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ON THE RELATIONSHIP BETWEEN ARRAY PERFORMANCE AND THE EXISTENCE--ETC(U)

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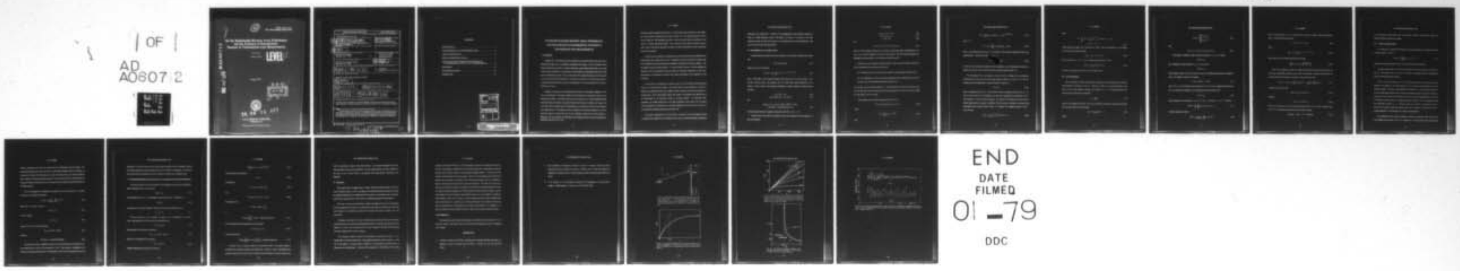

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On the Relationship Between Array Performance and the Existence of Deterministic Features in Transmission-Loss Measurements

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Acoustics Division

August 1978

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CONTENTS

ON THE RELATIONSHIP BETWEEN ARRAY PERFORMANCE AND THE EXISTENCE OF DETERMINISTIC FEATURES IN TRANSMISSION-LOSS MEASUREMENTS

I. Introduction

Suppose that a transmission-loss plot possesses a pronounced feature that clearly has a deterministic origin, e.g., a well-defined convergence-zone spacing. The very existence of this feature implies that the effect of random temperature (sound-speed) fluctuations in the ocean on the acoustic signal must, in some sense, be small because a strong effect would tend to wash out any deterministic structure. On the other hand, these random fluctuations are an important source of array performance degradation. Therefore, the existence of a deterministic feature in a transmission-loss plot places a limit on the degradation due to random fluctuations of the performance of an array.

Suppose this concept can be quantified and the limit on performance degradation turns out to be meaningful. For example, suppose that the existence of a convergence-zone structure implies that the effect of the random fluctuations is so small that they cannot significantly affect array performance. In addition, suppose the effect of the fluctuations increases with range. One could then make inferences about the performance of a hypothetical array from transmission-loss measurements. If convergence zones are observed at a particular frequency out to some maximum range one would know that an array collecting a signal from a source located at a range less than the maximum and radiating at that frequency would not have its performance

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limited by random temperature fluctuations. Its performance might be limited by other effects, e.g., array distortion, deterministic multi-paths, eddies, etc., but random fluctuations could at least be ruled out. The important point here is that one would reach this conclusion as the result of a modest experimental effort. The involved and costly spatial coherence measurements which would directly determine the effect of random fluctuations on array performance would not be necessary.

This idea that one might learn something about array performance from transmission-loss measurements may initially seem curious. Transmission loss and array gain are usually measured by different experimental groups and modelled using different theoretical analyses. They are separate terms in the sonar equation. It must be evident, however, that both quantities are measures of the same physical process and cannot be completely independent of each other. The question to be answered is whether their implicit relationship can be exploited in some useful way.

In this report we investigate this question using a multi-component, phenomenological model of the received acoustic signal. This simple model is only intended as an example to illustrate the concepts involved and to suggest the type of analysis that would be required for a detailed study. For the deterministic feature we consider a convergence zone defined here as an enhancement in the statistical average of the signal intensity. The particular multi-component we consider implies that if the single hydrophone which detects the convergence zone is replaced by a horizontal array, orientated broadside, this array will have a gain that is not reduced by random fluctuations by more than 3 dB.

The report is organized into six brief sections. In Section II the multi-component model is discussed and a definition of a convergence zone is given. Section III develops a criterion for

satisfactory array performance. In Section IV the significance of this criterion is discussed in terms of a single-component model of the signal. In Section V we return to the multi-component model and relate the existence of a convergence zone to array performance. Section VI contains some concluding remarks.

II. The Definition of a Convergence Zone

We shall assume the signal received at the position $\vec{x}$ and at time t is given by the expression

$$s(\vec{x},t) = Re\ p(\vec{x},t), \tag{2.1}$$

where p is a sum of components

$$p(\vec{x},t) = \frac{1}{r^{1/2}} \sum_{\alpha} A_{\alpha}\, e^{i(q_{\alpha}r - \omega t + \phi_{\alpha}(\vec{x},t) - \phi_{o})}. \tag{2.2}$$

Here r is the range, ω is the angular frequency of the source, and ϕ_o is some constant, unobservable, reference phase. The quantities $\phi_{\alpha}(\vec{x},t)$ result from random fluctuations in the medium. They are taken to be statistically independent, Gaussian random variables having the moments

$$<\phi_{\alpha}(\vec{x},t)> = 0, \tag{2.3}$$

$$<\phi_{\alpha}(\vec{x},t)\phi_{\alpha'}(\vec{x},t)> = \delta_{\alpha\alpha'}\Phi_{\alpha}^{2}(r), \tag{2.4}$$

and

$$<[\phi_{\alpha}(x,y,z,t) - \phi_{\alpha'}(x,y',z,t)]^2> = \Phi_{\alpha}^{2}(r) + \Phi_{\alpha'}^{2}(r) - 2\delta_{\alpha\alpha'}\Phi_{\alpha}^{2}(r) + 2\delta_{\alpha\alpha'}\Phi_{\alpha}^{2}(r)S_{\alpha}(y-y';r). \tag{2.5}$$

This last equation holds for propagation along the x-axis with $r >> |y - y'|$.

It follows from rather general considerations that the normalized structure functions S_{α} have the properties

$$S_\alpha\,(y;r) = S_\alpha\,(-y;r), \tag{2.6}$$

$$0 \leqslant S_\alpha\,(y;r) \leqslant 1, \tag{2.7}$$

$$S_\alpha\,(0;r) = 0, \tag{2.8}$$

and

$$S_\alpha\,(y;r) \leqslant S_\alpha\,(y';r) \text{ for } |y| \leqslant |y'|. \tag{2.9}$$

Since we will be primarily concerned with reception at a particular range, the dependence of Φ_α and S_α on r will be suppressed in much of what follows. We have already suppressed any dependence on source depth, receiver depth, and source frequency.

There are two, not necessarily mutually exclusive, ways in which the signal intensity may have an enhanced value at a particular range point:

(1) The phases in Eq. (2.2) are such that a number of the components coherently add.

(2) The amplitudes A_α have a strong range dependence and an enhancement is the result of one or more of these amplitudes assuming a large value.

In this paper we will consider possibility (1). The amplitudes therefore play a secondary role. We will take them to be non-random quantities which depend on source and receiver depths but only weakly, if at all, on range.

The average or mean intensity associated with Eq. (2.1) is

$$I \equiv \langle p^*\,(\vec{x},t)\,p(\vec{x},t)\rangle = J + K, \tag{2.10}$$

where

$$J = \frac{1}{r} \sum_\alpha A_\alpha^2\,(1 - e^{-\Phi_\alpha^2}), \tag{2.11}$$

and

$$K = \frac{1}{r} \sum_{\alpha\alpha'} A_\alpha A_{\alpha'} \cos\left[r(q_\alpha - q_{\alpha'})\right] e^{-\frac{1}{2}(\Phi_\alpha^2 + \Phi_{\alpha'}^2)} \tag{2.12}$$

As $\Phi_\alpha \rightarrow 0$,

$$I \rightarrow I_o \equiv \frac{1}{r} \sum_{\alpha\alpha'} A_\alpha A_{\alpha'} \cos\left[r(q_\alpha - q_{\alpha'})\right]. \tag{2.13}$$

Here I_o is the deterministic intensity, i.e., the intensity which would be measured if there were no fluctuations. On the other hand, [illegible] $\Phi_\alpha >> 1$,

$$I \rightarrow \text{[illegible]} A_\alpha^2. \tag{2.14}$$

In this case the individual components add incoherently, all the deterministic structure is washed out, and one has the random multi-path situation discussed in Ref. (1).

We will assume that a convergence zone exists when a number of the components coherently add in such a way that the average intensity is greater by a factor γ (>1) than the average intensity corresponding to complete incoherent addition, i.e.,

$$I \geqslant \gamma R, \tag{2.15}$$

where R is defined by Eq. (2.14). For the present we will not assign a numerical value to γ. In order for this criterion to be meaningful, one must have a technique for determining I and R. The intensity is, of course, directly measured. The method for estimating R from the experimental measuremtents is basically no different from the common technique of parameterizing transmission loss curves by a simple $10 \log_{10} r + A$ relation. For example, from Eqs. (2.10) — (2.12) we have

$$rR(r) = \frac{1}{\Delta r} \int_{r-\Delta r/2}^{r+\Delta r/2} dr' r' I(r') + X, \tag{2.16}$$

where

$$X = \sum_{\substack{\alpha\alpha' \\ \alpha \neq \alpha'}} A_\alpha A_{\alpha'} \frac{1}{\Delta r} \int_{r-\Delta r/2}^{r+\Delta r/2} dr' \cos\left[r' \left(q_\alpha - q_{\alpha'}\right)\right]$$

$$\times\, e^{(\Phi_\alpha^2 (r') + \Phi_{\alpha'}^2 (r'))/2} \tag{2.17}$$

Simple arguments suggest Φ_α^2 is proportional to range. With this dependence it is not difficult to see that X will be small provided

$$\Delta r |q_\alpha - q_{\alpha'}| >> 1,\ \alpha \neq \alpha'. \tag{2.18}$$

For our purposes $|q_\alpha - q_{\alpha'}| \gtrsim 1/$(a convergence-zone spacing). Hence,

$$rR(r) \simeq \frac{1}{\Delta r} \int_{r-\Delta r/2}^{r+\Delta r/2} dr' r' I(r') \tag{2.19}$$

provided the integration includes several convergence zones.

III. Array Performance

We will consider a linear, horizontal array consisting on N uniformly spaced, omnidirectional elements as illustrated in Fig. (1). The array is taken to be oriented broadside to the source. The ith array element is located at the position (x, y_i, z). The spacing between adjacent elements Δ is given by the expression

$$\Delta = \frac{L}{N-1}, \tag{3.1}$$

where L is the length of the array. If the array is located in an isotropic, incoherent noise field, the array gain is given by the expression

$$AG = 10 \log_{10} F(N), \tag{3.2}$$

where

$$F(N) = \frac{\sum_{i,j=1}^{N} \Gamma\ (y_i,\ y_j)}{\sum_{i=1}^{N} \Gamma\ (y_i,\ y_i)}, \tag{3.3}$$

with

$$\Gamma(y_i,\ y_j) = \langle p^*(x,y_i,z,t)\ p\ (x,y_j,z,t)\rangle. \tag{3.4}$$

If the signal is completely coherent across the aperture, $\Gamma\ (y_i,\ y_j) = \Gamma$, giving

$$F(N) = N.$$

For a completely incoherent signal, $\Gamma\ (y_i,\ y_j) = \delta_{ij} I$ so that

$$F(N) = 1,$$

and the signal-to-noise ratio of the array is the same as the signal-to-noise ratio of a single element. If we define, for fixed Δ, the quantity

$$f(N) = F(N+1) - F(N), \tag{3.5}$$

then $f(N) = 1$ for a completely coherent signal and $f(N) = 0$ for a completely incoherent signal. As a criterion for satisfactory array performance we will arbitrarily take

$$f(N) \geqslant \frac{1}{2}. \tag{3.6}$$

For the model we are considering, $\Gamma\ (y_i,\ y_j) = \Gamma\ (|y_i - y_j|, 0)$ and $\Gamma\ (y_i,\ y_i) = I$. Therefore

$$F(N) = \frac{2}{NI} \sum_{i=0}^{N} (N-i)\ \Gamma\ (\Delta\ i,\ 0) - 1. \tag{3.7}$$

From this equation we obtain

$$f(N) = \frac{2}{N(N+1)I} \sum_{i=1}^{N} i \Gamma\ (\Delta\ i,\ 0). \tag{3.8}$$

Since we are assuming $r >> L$, we will ignore the variation in range as one moves along the array. A simple calculation then gives

$$\Gamma(\Delta i, 0) = I - J(\Delta i), \tag{3.9}$$

where

$$J(\Delta i) \equiv \frac{1}{r} \sum_{\alpha} A_{\alpha}^{2} \left(1 - e^{-\Phi_{\alpha}^{2} S_{\alpha}(\Delta i)}\right). \tag{3.10}$$

The criterion (3.6) for coherent summations now reads

$$\sum_{i=1}^{N} iJ(\Delta i) \leqslant \frac{1}{2} I \left[\frac{N(N+1)}{2}\right]. \tag{3.11}$$

So far we have defined a criterion for satisfactory array performance and have cast this criterion into a particularly convenient form. Now we would like to consider the restriction the criterion places on the array gain. From Eqs. (3.8) and (3.9) we have

$$f(N-1) - f(N) = \frac{2}{(N+1)I} \left[J(\Delta N) - \frac{2}{N(N-1)} \sum_{i=1}^{N-1} iJ(\Delta i)\right]. \tag{3.12}$$

It follows from Eq. (2.9) that

$$J(\Delta N) \geqslant J(\Delta i) \text{ for } N \geqslant i. \tag{3.13}$$

Therefore,

$$f(N-1) - f(N) \geqslant 0. \tag{3.14}$$

If $f(N) \geqslant 0.5$, then $f(N-1) \geqslant 0.5$ and, by iteration, $F(N) > 0.5N$. Note, we always have $F(N) \leqslant N$. The coherent-summation criterion therefore implies

$$10 \log_{10} N - 3 \text{ dB} < AG \leqslant 10 \log_{10} N. \tag{3.15}$$

i.e., the array gain cannot differ from the ideal gain, $10 \log_{10} N$, by more than 3 dB if the coherent-summation criterion is satisfied.

IV. A Single-Component Signal

It is instructive to consider the limitations on array performance for a signal consisting of a single component. We have

$$F(N) = \frac{2}{N} \sum_{i=0}^{N} (N-i)\, e^{-\Phi^2 S(\Delta i)} - 1, \tag{4.1}$$

where Φ is the strength parameter and S is the normalized structure function.

The general dependence of the structure function on horizontal separation is illustrated in Fig. (2). The tangent lines at $y=0$ and $y=\infty$ intersect at a separation which is approximately equal to the phase correlation length L_ϕ.

In Fig. (3) we have plotted $F(N)$ vs N for various values of Φ. The curves are only intended to be suggestive; the precise behavior of $F(N)$ depends on the detailed form of the structure function. The scale is determined by the number $N_\phi = L_\phi/\Delta$, which corresponds to an array length approximately equal to the phase correlation length. For small Φ, the fluctuations have a modest effect on the behavior of $F(N)$, even for array lengths large in comparison to the phase correlation length. For larger values of Φ, however, $F(N)$ only weakly increases with increasing N, unless $L \lesssim L_\phi$. The dots on the curves indicate the maximum value of N for which the coherent-summation criterion, Eq. (3.6), is satisfied. This maximum value is strongly dependent on Φ, decreasing very rapidly with increasing Φ.

The significance of the coherent-summation criterion is illustrated in Fig. (4) where the curve defined by the equation $f(N) = 0.5$ divides the $N-\Phi$ quarter-plane into a region where

coherent summation occurs and one where the array incoherently sums the signal. The coherent-summation region is the sum of the two sub-regions labelled I and II in the figure. In sub-region I the effect of the fluctuations is so weak that it does not matter if the array length is large compared to the phase-correlation length. In sub-region II, however, the fluctuations are strong and coherent summation occurs only because the array length is somewhat less than the correlation length.[2]

The two sub-regions are distinguished by whether or not Φ is less than Φ_{LIM}. To determine Φ_{LIM}, we consider the expression

$$f(N) = \frac{2}{N(N+1)} \sum_{i=1}^{N} i e^{-\Phi^2 S(\Delta i)}. \tag{4.2}$$

Since $S(\Delta\, i) \leqslant 1$ for all i, we have

$$f(N) \geqslant e^{-\Phi^2} \tag{4.3}$$

for all N. Hence,

$$e^{-\Phi^2} \geqslant \frac{1}{2} \tag{4.4}$$

implies $f(N) \geqslant 0.5$. We therefore have

$$\Phi_{LIM} = (ln 2)^{1/2} \simeq 0.83. \tag{4.5}$$

Moreover,

$$\Phi \leqslant \Phi_{LIM} \rightarrow \text{coherent summation}. \tag{4.6}$$

We were able to derive a sufficient condition for coherent summation by setting the structure function equal to unity in the expression for $f(N)$. This condition is dependent on the criterion for coherent summation but it is independent of both N and the detailed nature of the

fluctuations. In the next section we will use the same technique to derive a sufficient condition for coherent summation when the signal is the sum of a number of components. We will then show this condition is satisfied if the array is located in the region of a convergence zone.

V. The Relationship Between the Existence of a Convergence Zone and Array Performance

We are now ready to relate the existence of a convergence-zone structure to array performance. Referring to Eq. (3.10), we have

$$J(\Delta i) \leqslant J, \tag{5.1}$$

with J defined by Eq. (2.11). This inequality follows from Eq. (2.7). Therefore, if

$$J \leqslant \frac{1}{2} I, \tag{5.2}$$

the condition (3.11) will be satisfied. Using Eq. (2.10) we can rewrite Eq. (5.2) as

$$K - J \geqslant 0. \tag{5.3}$$

We now assume the array is located in a region where a convergence-zone structure exists. According to Eqs. (2.10) and (2.15) we therefore have

$$J + K \geqslant \gamma R. \tag{5.4}$$

This condition can obviously be rewritten as

$$(K - J) - (\gamma R - 2J) \geqslant 0. \tag{5.5}$$

Now Eq. (5.5) implies Eq. (5.3) provided

$$\gamma R - 2J \geqslant 0. \tag{5.6}$$

Using the definitions of R and J, Eq. (5.6) reads

$$\frac{1}{r}\sum_{\alpha} A_{\alpha}^{2}\left(\gamma - 2 + 2e^{-\Phi_{\alpha}^{2}}\right) \geqslant 0. \tag{5.7}$$

This condition will be satisfied if

$$\gamma \geqslant 2. \tag{5.8}$$

Summarizing,

$$K - J \geqslant 0 \rightarrow f(N) \geqslant \frac{1}{2} \tag{5.9}$$

and

$$I \geqslant \gamma R;\ \gamma \geqslant 2 \rightarrow K - J \geqslant 0. \tag{5.10}$$

We therefore have

$$I \geqslant 2R \rightarrow f(N) \geqslant \frac{1}{2},$$

or

$$10 \log_{10}\left|\frac{I}{R}\right| > 3\ dB \rightarrow \text{coherent summation.} \tag{5.11}$$

If we had defined coherent summation by the inequality

$$f(N) \geqslant \frac{1}{\rho};\ \rho > 1, \tag{5.12}$$

we would have found

$$10 \log_{10}\left|\frac{I}{R}\right| \geqslant 10 \log_{10}\left|\frac{\rho}{\rho-1}\right| \rightarrow \text{coherent summation.} \tag{5.13}$$

With Eq. (5.11), we end our analysis of this particular model. This equation implies a horizontal array oriented broadside and operating in a region in which a convergence-zone structure exists will have a gain that is not reduced from the ideal gain, $10 \log_{10} N$, by more than

3 dB. It is important to realize it is the mean intensity, i.e., the intensity averaged over the random fluctuations which is being considered. The unaveraged intensity may have variations on the order of two or more without a convergence zone being present (according to our definition).

VI. Discussion

This analysis does not suggest that in a region where the average intensity is small, an array will perform poorly. In fact, if the effects of the random fluctuations increase with range, our analysis implies that for all ranges less than the range of a convergence zone, the fluctuations will be so weak that they will not result in a significant degradation of performance.

There are, of course, many situations in which convergence zones are not pronounced. For such situations this analysis is not particularly relevant and one would want to find some other indicator of the deterministic aspects of the process with which to correlate with array performance.

In principle, the relation (5.11) can be checked experimentally by towing a source along a radial track normal to an array and simultaneously measuring array gain and transmission loss. Suppose it is correct. One would then have a simple, inexpensive technique for determining how well a proposed array is likely to perform.

As an example, consider the results of the experiment conducted by Guthrie, *et al.*,[3] during 1969 in the North Atlantic Basin. They measured transmission loss vs range for a 14-Hz and 111-Hz signal to a maximum range of 2800 km. The transmission loss plots of Fig. (5) summarize their measurements. These plots were generated by performing a 7-km running

average of the received intensity. For both frequencies, consecutive convergence zones are evident out to long ranges. Although the 7-km running average tends to approximate a statistical average, what is plotted is really not the statistically averaged intensity. (A time series of the intensity at each range point was not averaged.) However, the convergence-zone structure is so apparent and so obviously a characteristic of the deterministic problem that it is not difficult to believe it would persist had a statistical average been performed. For either plot we have indicated by an arrow the last distinct convergence zone which has an intensity greater by a factor of 2 than the intensity averaged over a 300-km interval centered at the zone location. For the 14-Hz signal this zone is located at about 2300 km and for the 111-Hz signal it is located at about 1100 km. If Eq. (5.11) is correct, we would conclude that random ocean variability does not limit the ability of a horizontal array, orientated broadside and operating in this environment, to coherently sum a 14-Hz signal out to a range of 2300 km and a 111-Hz signal out to a range of 1100 km even if the length of the array is larger than the signal correlation distance.

Acknowledgements

This report was conceived and written during a two-week period in the fall of 1977 in an attempt to clarify, in the author's mind, a point of view first expressed to him by R. Fitzgerald and J. Shaffer.

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2. For a discussion of sub-region II within the context of a model in which the random fluctuations are due to internal waves, see R. D. Dashen, *et al.*, "Limits on Coherent Processing Due to Internal Waves," Stanford Research Institute Technical Report JSR-76-14 (1977).

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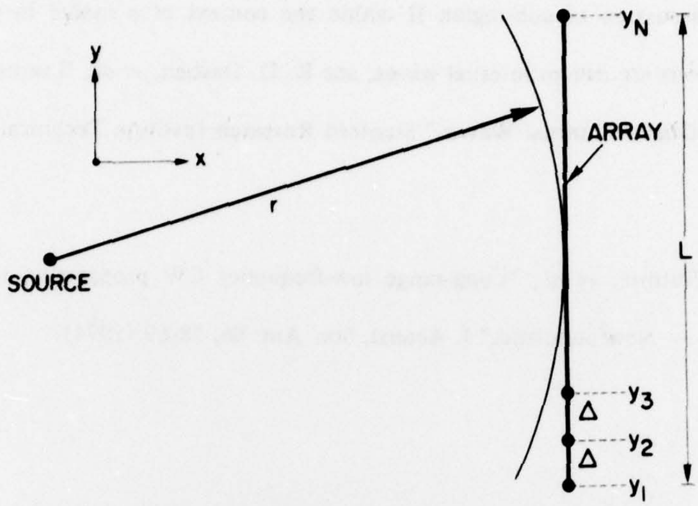

Fig. (1) — Array geometry. The array is of length L and consists of N elements spaced Δ apart. It is parallel to the y-axis so that the ith array element is located at (x, y_i, z). It is oriented broadside to the source. The source is a distance r away from the array where $r >> L$. The slight variation of the range along the array is ignored in the calculation of the array gain.

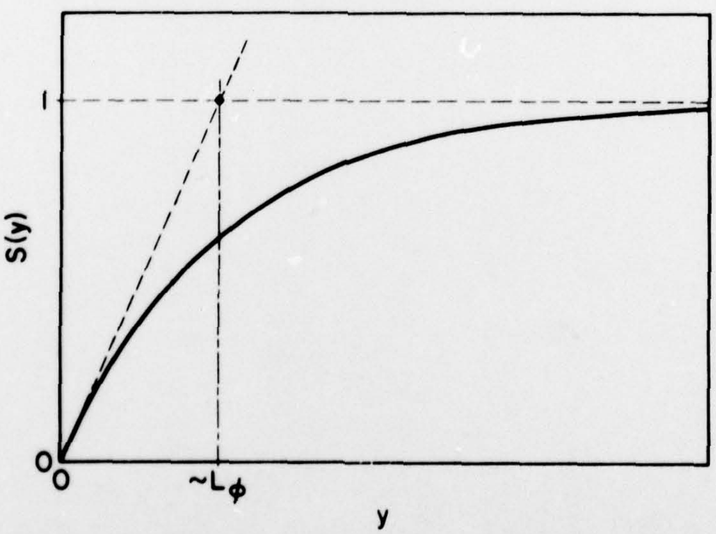

Fig. (2) — The characteristic dependence of the normalized structure function on horizontal separation y. The lines tangent to the curve at $y=0$ and $y=\infty$ intersect at a separation which is approximately equal to the phase correlation length L_ϕ.

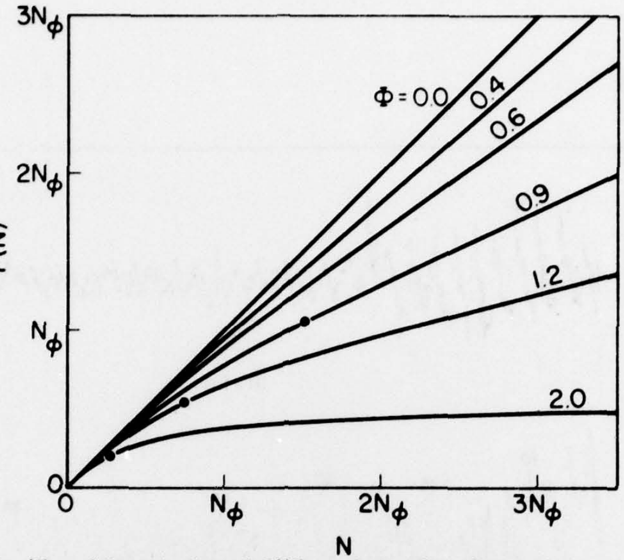

Fig. (3) — Schematic plots of $F(N)$ vs. the number of array elements N for various values of the strength parameter. The number N_ϕ is defined as the ratio of the phase correlation length to the interelement spacing. The dots indicate the maximum value of N for which the coherent-summation criterion is satisfied.

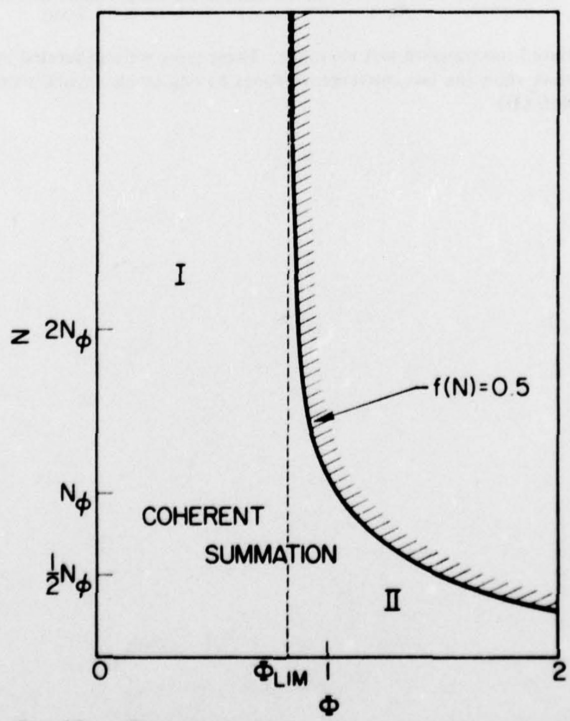

Fig. (4) — The region where coherent summation occurs. This region is divided into two subregions, I and II, according to whether Φ is less than or greater than Φ_{LIM}.

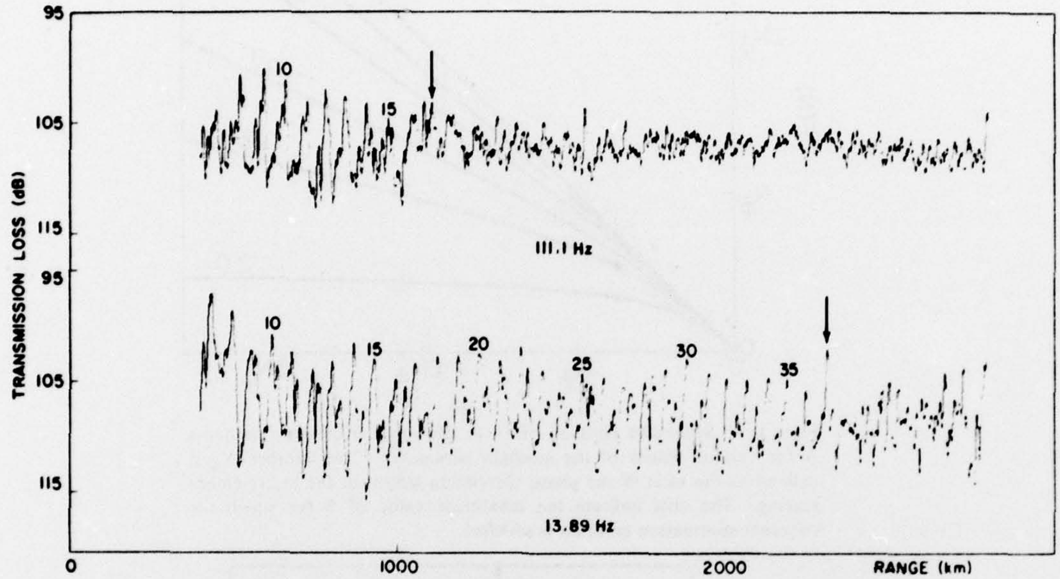

Fig. (5) — Plots of measured transmission loss vs. range. These plots were generated by averaging the received intensity over 7 km. The arrows show the last convergence zones having levels approximately 3 dB above the background levels. (Adopted from Ref. (3)).